Cone Volumes
Teacher Sheet

Introduction

The aim of this activity is to have students work out the volume of a cone using a variety of different known measurements. This involves application of Pythagoras' Theorem, and right-angled triangle Trigonometry.

Then the students progress to finding the optimum apex angle to maximise the volume of a cone for a given slant height. **No knowledge of calculus** is required – the process is completed using numeric Graph Analysis tools.

This activity is designed for students aged 12 to 15, both as a consolidating activity of existing skills and preparing the conceptual way for future optimisation problems.

Although the screenshots in this document are taken from a colour screen Nspire CX, the activity works just as well on a greyscale Nspire handheld. However, OS 3.0.2 or later is required.

Problem 1 – Introductions and Assumptions

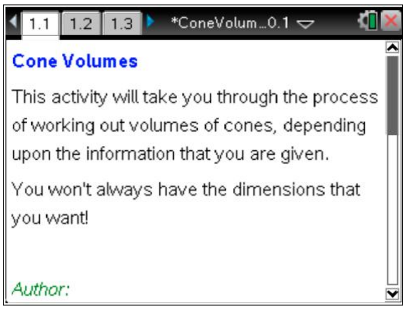
The first 11 pages introduce the vocabulary that will be used, and checks the students' existing knowledge and ability to apply Pythagoras (pages 1.6 and 1.7) and Trigonometry (pages 1.8 to 1.11). In order to answer the questions on these pages, students will most likely need to draw their own diagrams on their page, as well as use the Scratchpad for the evaluation of their calculations. *The Scratchpad Calculator can be accessed by pressing (scratchpad key), and afterwards press (esc) to return to the document.*

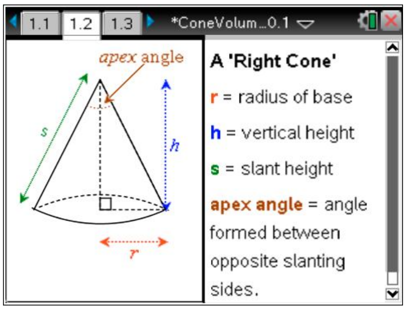
In the process of working out the numerical answers, students may report that they obtain a similar, but not exactly the same decimal answers as those offered as options. This situation can arise from using, say, 3.14 instead of the exact value of π, or by not using the full decimal value of a previous answer. If this occurs, students should be **shown by the teacher** how to use the Scratchpad to generate answers that not only use π, but that also do not suffer from rounding errors.

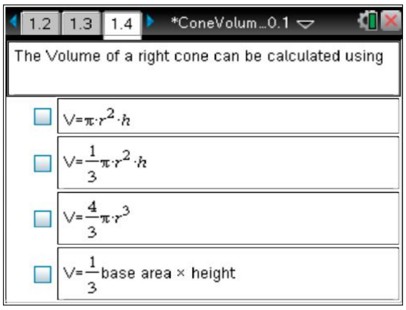
Students also need to be aware of whether their Scratchpad settings are in **Degrees or Radians**. It can be left to the teacher to decide whether to forewarn the students of this, or to let them have the learning experience of realising this themselves.

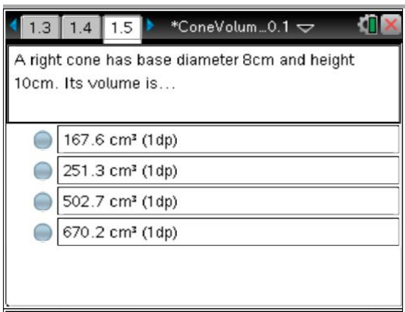
The question pages have been designated to be 'Self-Check', so students should press (ctrl) then ▲ to verify if they are correct, after each one.

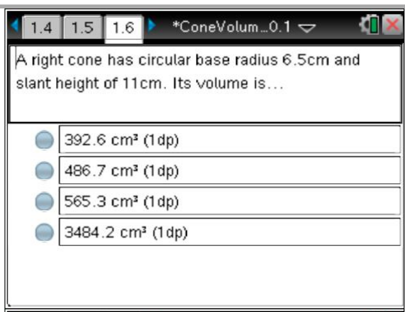
Students should note that page 1.4 has square option boxes, meaning that more than one option is correct, and that they should select **all** the correct formulae in order for it to be marked correct.

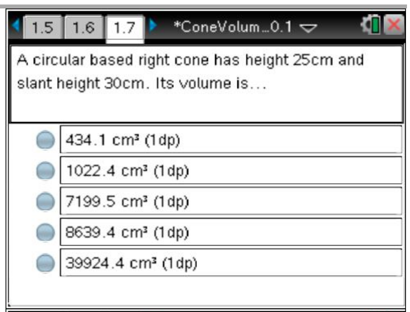


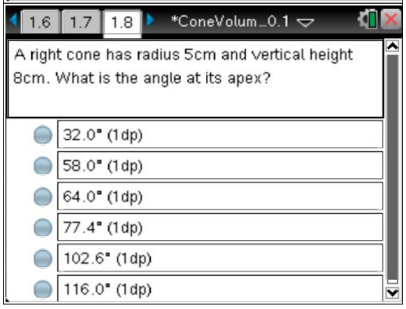


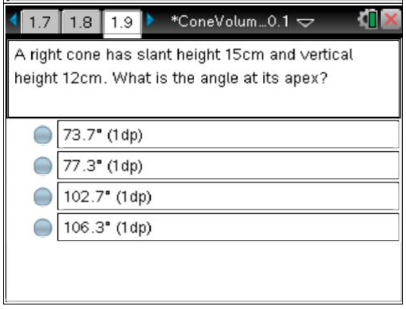


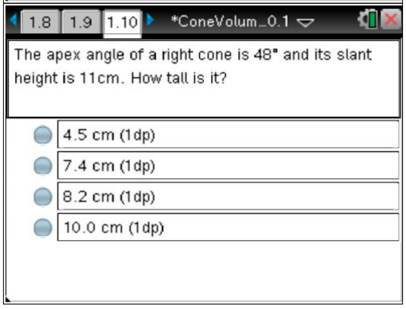


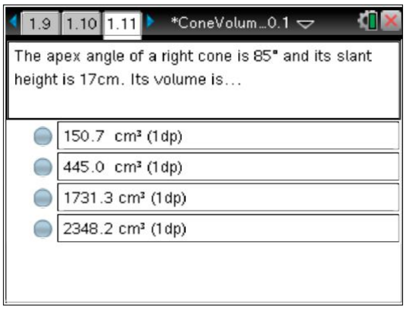








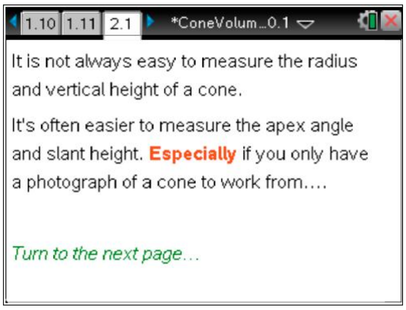


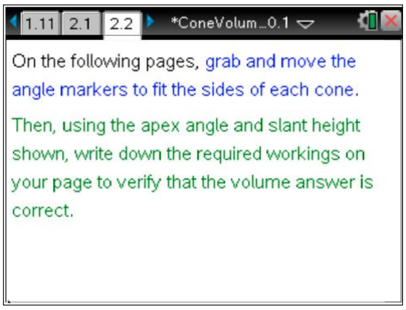


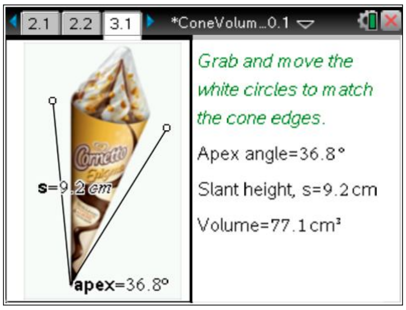
The incorrect, distractor answers on each page are all generated from making 'Classic Mistakes' such as adding instead of subtracting when using Pythagoras' Theorem, using the incorrect trigonometric ratio, not halving the apex angle when dealing with the angle inside the right-angled triangle, etc.

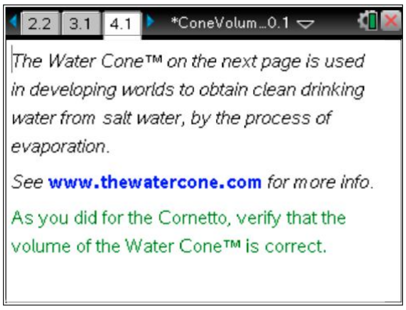
Problems 2, 3 and 4 – Focusing on the Apex Angle in Context

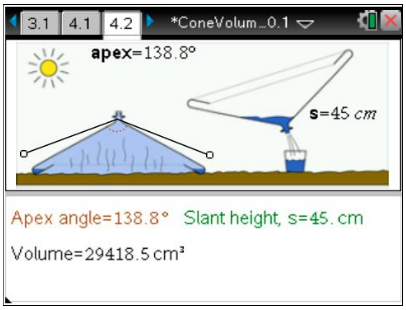
Pages 2.1 and 2.2 introduce problems 3 and 4. On pages 3.1 and 4.2, students should grab the white circles to align the segments with the sides of the cone, and thus read off the accurate apex angle.







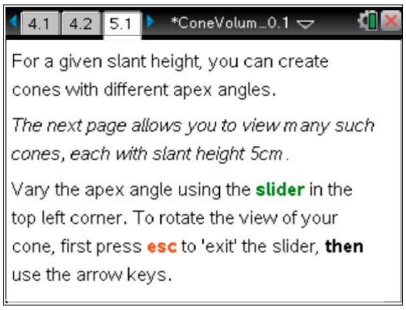


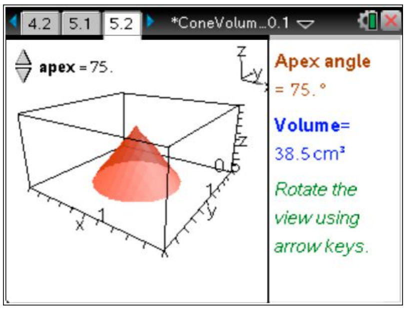


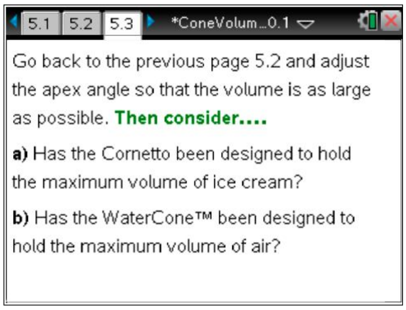
Problem 5 – Optimising the Cone Volume

Page 5.2 is important for students to experiment with, to appreciate that the maximum volume comes when the apex angle is around 110°. When the apex angle is set to 0° or 180°, are the students surprised by what they see (or don't see!) ?

Page 5.3 could be used for class discussion about why the Cornetto and WaterCone™ were not designed to maximise the volume of their cones.

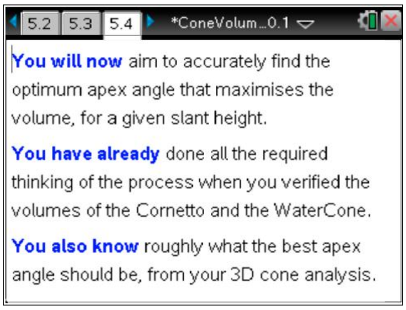


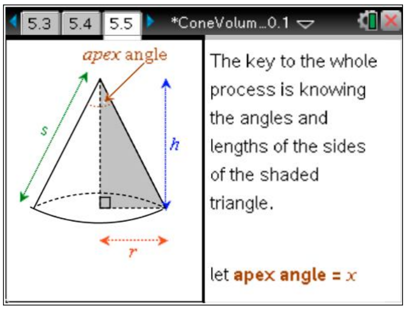


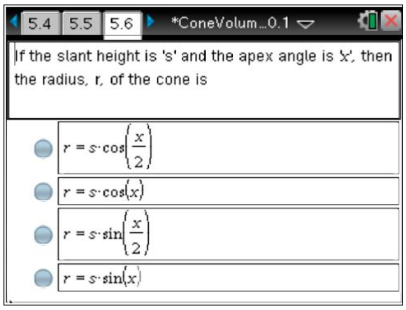


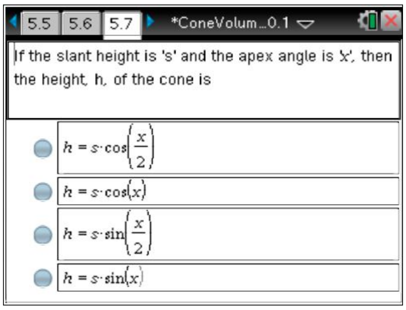
Page 5.4 introduces the task of optimising the volume, whilst pages 5.5 to 5.8 take students step-by-step towards deriving the algebraic function for the volume in terms of slant height, s, and apex angle, x.

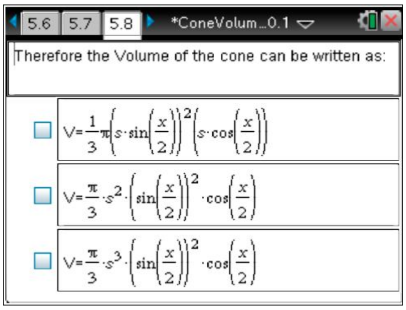
Page 5.8 requires students to correctly identify **all** of the valid algebraic versions (more than one is correct)

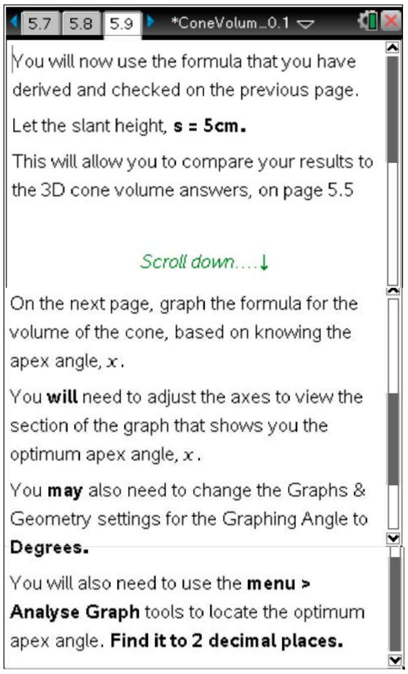






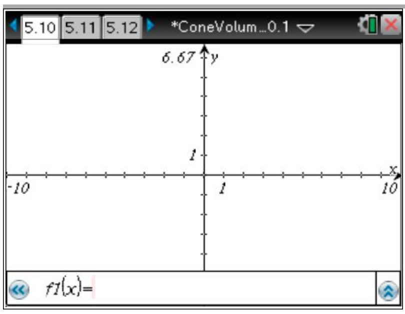


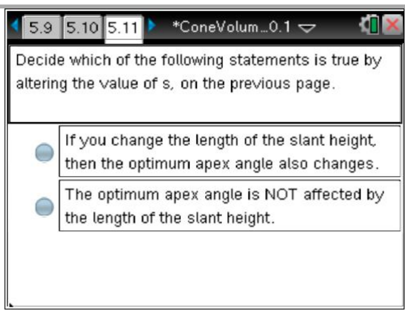


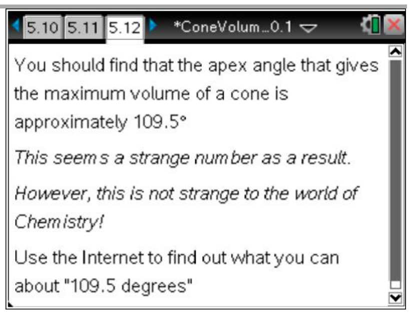


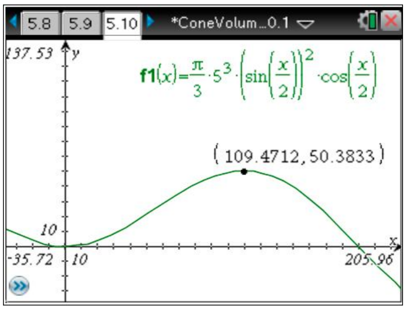
When analysing the graph on page 5.10, students need to know how to use the 'maximum' option.

To display the answer to 2 decimal places, one way is to move over the x co-ordinate of the maximum turning point and then press the ⊕ key a few times.







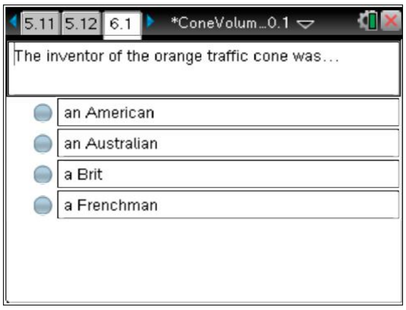


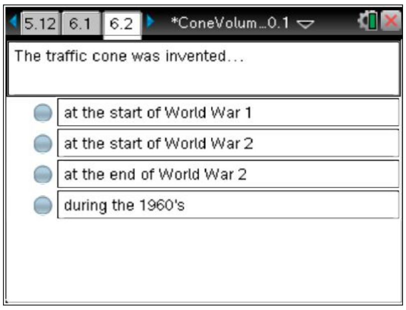
The completed graph is shown on the left, for when the slant height, s = 5 cm.
The maximum volume of 50.8333.. cm^3 comes when the apex angle is 109.4712.... degrees.

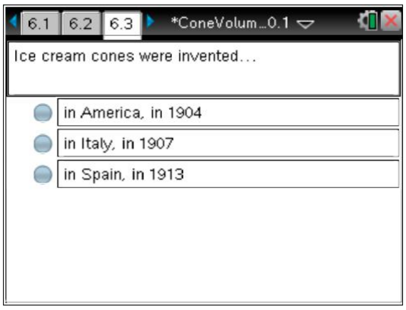
Problem 6 – Light Hearted Ending, with Extension Task.

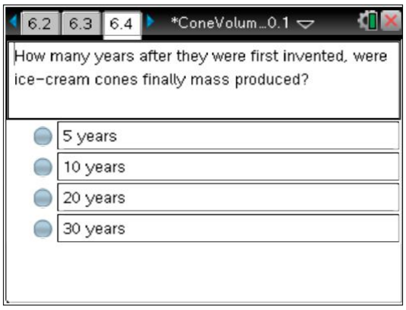
Pages 6.1 to 6.4 are meant to be a light-hearted ending to the task.

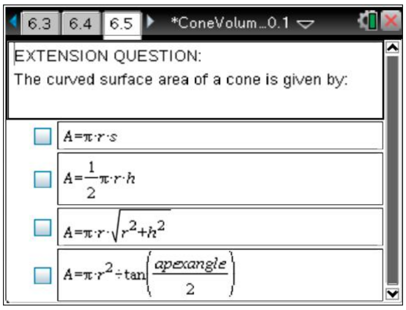
Page 6.5 can be used as an extension task for able students who finish early to attempt to solve. Again, more than one correct answer exists on this page. Once the correct answers have been identified, students could then embark upon deriving these formulae from scratch – this might take another significant chunk of time.











Acknowledgements

Images used in this activity were sourced from the following internet pages on 18 June 2011.

Page 3.1
http://www.fdin.org.uk/wordpress/wp-content/uploads/Cornetto_Enigma_VanChoc_Wrap.jpg

Page 4.2
http://greentechfreedom.com/wp-content/uploads/2010/11/funktion1600.jpg

